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ZAMONAVIY TA'LIMDA ISLOHOTLAR VA TEXNIK TAFAKKUR

ilmiy jurnali

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“ZAMONAVIY TA'LIMDA ISLOHOTLAR VA TEXNIK TAFAKKUR”

ilmiy jurnali

Mazkur ilmiy to'plamda “Zamonaviy ta'limda islohotlar va texnik tafakkur” ilmiy jurnaliga taqdim etilgan ilmiy maqolalar, tezislar hamda ilmiy-tadqiqot natijalari o'rin olgan. Jurnal zamonaviy ta'lim tizimini rivojlantirish, ta'lim jarayoniga innovatsion pedagogik va axborot texnologiyalarini joriy etish, ta'lim mazmuni va metodlarini takomillashtirish, shuningdek, pedagogika, texnika va muhandislik yo'nalishlarining o'zaro integratsiyasini rivojlantirishga qaratilgan ilmiy izlanishlarni yoritishga ixtisoslashgan.

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NOCHIZIQLI GIPERBOLIK SISTEMALAR UCHUN LYAPUNOV MA'NOSIDA TURG'UN OQIMGA QARSHI PARCHALANUVCHI AYIRMALI SXEMA

Xaydarova Ziyodaxon Azimjon qizi ¹

¹ Toshkent menejment va iqtisodiyot instituti Farg'ona filiali.

Annotatsiya: Ushbu maqolada bir o'lchovli, o'zgaruvchan koeffitsientli, nol tartibli bog'lanish hadiga ega hamda dissipativ chegaraviy shartlarga bo'ysunuvchi chiziqli giperbolik balans qonunlari sistemasi o'rganiladi. Tadqiqotning maqsadi — uzluksiz aralash masalaning Lyapunov ma'nosidagi eksponensial turg'unligi uchun yetarli shartni aniqlash va shu shartga mos keluvchi, sonli yechimni hisoblashda ham eksponensial turg'unlikni saqlovchi ayirmali sxema qurishdan iborat. Tadqiqot og'irlangan kvadratik Lyapunov funksiyasi usuliga asoslanadi: xarakteristik tezliklarga bog'liq eksponensial og'irlik funksiyalari kiritiladi va chegaraviy shartlarning dissipativligi $|\Delta R \Delta^{-1}| < 1$ matritsaviy sharti orqali ifodalanadi. Chegaraviy shartlarning dissipativligi va bog'lanish matritsasining nomanfiy aniqlanganligi asosida uzluksiz masalaning L^2 -normada eksponensial turg'unligi to'liq isbotlanadi. Sonli yechim uchun oqimga qarshi parchalanuvchi (fractional-step) oshkor ayirmali sxema qurilgan. Diskret Lyapunov funksiyasi, bir qadamli kontraktsiya bahosi va xarakteristik tezliklarning Lipschitz uzluksizligiga asoslangan o'lchov (scaling) lemmasi yordamida, Kurant–Fridrixs–Levi (KFL) sharti va aniq ν -to'r sharti ostida sxemaning ℓ^2 -normadagi eksponensial turg'unligi isbotlangan. Diskret dissipativlik va bog'lanish hadining nomanfiyligi uzluksiz teoremaning tabiiy diskret analoglari sifatida alohida shart tarzida qo'yilgan. Nazariy natijalar sonli tajribalar bilan tasdiqlangan: shartlar bajarilganda diskret Lyapunov funksiyasi monoton kamayadi, shartlar buzilganda esa o'sish kuzatiladi. Olingan natijalar giperbolik tipdagi chegaraviy masalalar uchun turg'un sonli sxemalarni qurishda, xususan gidravlika, gaz dinamikasi va transport jarayonlari modellarida qo'llanilishi mumkin.

Kalit so'zlar: giperbolik balans qonunlari; Lyapunov turg'unligi; eksponensial turg'unlik; oqimga qarshi ayirmali sxema; diskret Lyapunov funksiyasi; KFL sharti; dissipativ chegaraviy shartlar.

Аннотация. В данной статье исследуется одномерная линейная гиперболическая система законов сохранения (балансовых законов) с переменными коэффициентами, слагаемым нулевого порядка (членом связи) и диссипативными граничными условиями. Цель работы — установить достаточное условие экспоненциальной устойчивости по Ляпунову непрерывной смешанной задачи и построить соответствующую разностную схему, сохраняющую экспоненциальную устойчивость численного решения. Анализ основан на взвешенном квадратичном функционале Ляпунова с экспоненциальными весами, зависящими от характеристических скоростей, при этом диссипативность граничных условий выражается матричным условием $|\Delta R \Delta^{-1}| < 1$. При выполнении этого условия диссипативности и неотрицательной определённости матрицы связи доказана полная экспоненциальная устойчивость непрерывной задачи в норме L^2 . Для численного решения построена явная противопотоковая (upwind) расщепляющаяся разностная схема. С помощью дискретного функционала Ляпунова, одношаговой оценки сжатия и леммы масштабирования, основанной на липшицевой непрерывности характеристических скоростей, доказана экспоненциальная устойчивость схемы в норме ℓ^2 при выполнении условия Куранта–Фридрихса–Леви (CFL) и явного условия на сетку. Дискретная диссипативность и неотрицательность дискретного члена связи введены как





естественные дискретные аналоги непрерывных предположений. Численные эксперименты подтверждают теоретическую скорость убывания и её нарушение при несоблюдении гипотез. Полученные результаты могут быть использованы при построении устойчивых численных схем для гиперболических краевых задач, в частности в гидравлике, газовой динамике и моделировании транспортных процессов.

Ключевые слова: гиперболические законы сохранения; устойчивость по Ляпунову; экспоненциальная устойчивость; противопотоковая разностная схема; дискретная функция Ляпунова; условие CFL; диссипативные граничные условия.

Abstract. This paper studies a one-dimensional linear hyperbolic system of balance laws with variable coefficients, a zero-order coupling term, and dissipative boundary conditions. A sufficient condition for L^2 -exponential stability of the continuous problem is proved in full. On this basis, an upwind fractional-step (splitting) explicit difference scheme is constructed for the numerical solution of the system. Exponential stability of the scheme's numerical solution in the ℓ^2 -norm is proved under an explicit mesh condition arising from the Lipschitz continuity of the characteristic speeds and under the Courant–Friedrichs–Lewy (CFL) condition. Numerical experiments illustrate the theoretical decay rate and its loss when the hypotheses are violated.

Keywords: hyperbolic balance laws; Lyapunov stability; exponential stability; upwind difference scheme; discrete Lyapunov function; CFL condition; dissipative boundary conditions.

1 Introduction

Hyperbolic balance laws describe gas dynamics, open-channel flow, and transport processes. For such systems, the stability of the steady state is a fundamental issue of both theoretical and engineering interest [1]. The exponential stability of a linearized hyperbolic system is, in general, not equivalent to the local exponential stability of the underlying nonlinear system: even for smooth initial data close to a steady state, trajectories of nonlinear conservation laws may develop shock-type discontinuities in finite time. For this reason, the stability of the linear system itself, together with the stability of its numerical discretization, must be established directly, without relying on indirect linearization arguments.

A natural and robust tool for such an analysis is the method of Lyapunov functions, adapted to the hyperbolic setting through weighted L^2 -type functionals [1, 2]. The construction of a Lyapunov function that simultaneously controls the continuous solution and admits a discrete analogue compatible with a numerical scheme is the central technical task of this paper.

The contributions of this paper are threefold:

1. We prove a sufficient condition for exponential stability, in the sense of Lyapunov, of a linear hyperbolic system of balance laws with variable coefficients and dissipative boundary conditions (Theorem 11), with an explicit, fully derived proof.
2. We construct an upwind fractional-step explicit difference scheme for the numerical solution of the system, satisfying the Courant–Friedrichs–Lewy (CFL) stability condition.

3. We prove exponential stability of the discrete solution in the ℓ^2 -norm (Theorem 24), via a discrete Lyapunov function, a one-step contraction estimate for the upwind transport step, and an explicit mesh condition derived from the Lipschitz continuity of the characteristic speeds.

The remainder of the paper is organized as follows. Section 2 states the mixed initial-boundary value problem. Section 3 proves the continuous exponential stability theorem. Section 4 introduces the difference scheme. Section 5 develops the discrete Lyapunov theory and proves the discrete stability theorem. Section 6 reports numerical experiments. Section 7 concludes.

2 Statement of the Problem

2.1 Linearization

Consider a nonlinear system of conservation laws linearized about a steady state $Y^*(x)$. Through the Riemann coordinates $v = \psi(Y) - \psi(Y^*)$, the linearized system takes the form

$$v_t + K(x)v_x + M(x)v = 0,$$

where K is the matrix of characteristic speeds and M is the linearized coupling matrix.

2.2 Canonical Form and the Mixed Problem

Write $v = \begin{pmatrix} v^I \\ v^{II} \end{pmatrix}$, $v^I = (v_1, \dots, v_m)^T$, $v^{II} = (v_{m+1}, \dots, v_n)^T$, and

$$K(x) = \begin{pmatrix} K^+(x) & 0 \\ 0 & -K^-(x) \end{pmatrix}, K^+ = \text{diag}(a_1, \dots, a_m), K^- = \text{diag}(a_{m+1}, \dots, a_n),$$

with $a_i \in C^1([0, L])$, $0 < a_{\min} \leq a_i(x) \leq a_{\max}$. The first m characteristic speeds are positive (rightward), while the remaining $n - m$ are negative (leftward). Let $s \in \mathbb{R}^{m \times (n-m)}$, $r \in \mathbb{R}^{(n-m) \times m}$ be constant matrices, and let

$$R = \begin{pmatrix} 0 & s \\ r & 0 \end{pmatrix}.$$

The system is equipped with the dissipative boundary conditions

$$v^I(0, t) = s v^{II}(0, t), v^{II}(L, t) = r v^I(L, t),$$

and the initial condition

$$v(x, 0) = \varphi(x), x \in [0, L].$$

Definition. [Exponential stability] Problem (1)–(6) is said to be *exponentially stable in the L^2 -norm* if there exist $\nu > 0$ and $c > 0$ such that, for every $\varphi \in L^2((0, L); \mathbb{R}^n)$,

$$|v(t)|_{L^2} \leq c e^{-\nu t} |\varphi|_{L^2}, t \geq 0.$$

3 Exponential Stability of the Continuous Problem

3.1 Weight Matrices and the Lyapunov Function

For $\nu > 0$ and $\mu_i > 0$, $i = 1, \dots, n$, define the weight matrices

$$\mu^+(v x) = \text{diag}(\mu_i e^{-\int_0^x \frac{\nu}{a_i(\sigma)} d\sigma})_{i \leq m}, \mu^-(v x) = \text{diag}(\mu_i e^{+\int_0^x \frac{\nu}{a_i(\sigma)} d\sigma})_{i > m},$$

$\mu = \text{diag}(\mu^+, \mu^-)$, and the Lyapunov function

$$V(t) = \int_0^L v^T |K(x)|^{-1} \mu(vx) v dx, |K| = \text{diag}(K^+, K^-).$$

Since $a_{\min} \leq a_i \leq a_{\max}$, we have $0 < \mu_i e^{-vL/a_{\min}} \leq \mu_i e^{\pm \int_0^x v/a_i} \leq \mu_i e^{vL/a_{\min}}$, whence

$$c_1 |v|_{L^2}^2 \leq V(t) \leq c_2 |v|_{L^2}^2, c_1 = \frac{\min \mu_i}{a_{\max}} e^{-vL/a_{\min}}, c_2 = \frac{\max \mu_i}{a_{\min}} e^{vL/a_{\min}}.$$

3.2 Dissipativity Condition

Over the set of positive diagonal matrices $D = \text{diag}(D_0, D_1)$, $D_0 \in \mathbb{R}^{m \times m}$, $D_1 \in \mathbb{R}^{(n-m) \times (n-m)}$, define

$$\rho_2(R) = (1 + \gamma) \inf_D |DRD^{-1}|_2, \gamma > 0.$$

In the theorem below the condition $\rho_2(R) < 1$ is used in its sufficient form: there exists $\Delta = \text{diag}(D_0, D_1)$ such that $|\Delta R \Delta^{-1}| < 1$.

Theorem. [Continuous stability] Assume:

1. there exists a positive diagonal matrix $\Delta = \text{diag}(D_0, D_1)$ such that $\delta_0 := 1 - |\Delta R \Delta^{-1}|^2 > 0$;
2. for all $x \in [0, L]$, $H(x, v) := |K|^{-1} \mu(vx) M(x) + M^T(x) |K|^{-1} \mu(vx) \geq 0$.

Then, for sufficiently small $v > 0$, problem (1)–(6) is exponentially stable in the L^2 -norm:

$$|v(t)|_{L^2} \leq \sqrt{c_2/c_1} e^{-vt/2} |\varphi|_{L^2}, t \geq 0.$$

Proof. Set $\chi(vx) := |K|^{-1} \mu(vx) K(x) = \text{diag}(\mu^+(vx), -\mu^-(vx))$. Taking the scalar product of (1) with $|K|^{-1} \mu v$, integrating over $x \in [0, L]$, and integrating by parts the transport term, using $\partial_x \mu^+(vx) = -v |K^+|^{-1} \mu^+(vx)$ and $\partial_x \mu^-(vx) = +v |K^-|^{-1} \mu^-(vx)$, yields

$$V'(t) = -vV(t) - [(\chi v, v)]_{x=0}^{x=L} - \int_0^L (Hv, v) dx = -vV(t) + W - \int_0^L (Hv, v) dx.$$

By Assumption 2, the last integral is nonnegative, hence $-\int_0^L (Hv, v) dx \leq 0$.

Using the boundary conditions (4)–(5), the boundary term is

$$W = ((r^T \mu^-(vL)r - \mu^+(vL))v_L^I, v_L^I) + ((s^T \mu^+(0)s - \mu^-(0))v_0^{II}, v_0^{II}).$$

Choosing $\mu^+(0) = D_0^2$, $\mu^-(0) = D_1^2$, and setting $x = D_0 v_L^I$, $y = D_1 v_0^{II}$, at $v = 0$ we obtain

$$W(0) = -((I - (\Delta R \Delta^{-1})^T (\Delta R \Delta^{-1})) \begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} x \\ y \end{pmatrix}) \leq -\delta_0 (|x|^2 + |y|^2),$$

by Assumption 1. Since $v \mapsto W(v)$ is continuous, there exists $v_0 > 0$ such that $W(v) \leq -\frac{\delta_0}{2} (|x|^2 + |y|^2) \leq 0$ for all $0 < v \leq v_0$.

Consequently, $V'(t) \leq -vV(t)$ for $0 < v \leq v_0$, so that $V(t) \leq e^{-vt} V(0)$. Combined with (10), this yields (12).

4 Upwind Fractional-Step Explicit Difference Scheme

4.1 The Scheme

Let $\Delta t = T/K$, $\Delta x = L/J$, $t^\kappa = \kappa \Delta t$, $x_j = j \Delta x$, $(v_i)^\kappa_j \approx v_i(t^\kappa, x_j)$, and $\rho_{ij} = (a_i)_j \Delta t / \Delta x$. The scheme consists of an upwind transport step

$$(u_i)^\kappa_j = (v_i)^\kappa_j - \rho_{ij}[(v_i)^\kappa_j - (v_i)^\kappa_{j-1}], i \leq m, j = 1, \dots, J, (u_i)^\kappa_j = (v_i)^\kappa_j + \rho_{ij}[(v_i)^\kappa_{j+1} - (v_i)^\kappa_j], i > m, j = 0, \dots, J-1,$$

followed by an explicit coupling step

$$(v_i)^{\kappa+1}_j = (u_i)^\kappa_j - \Delta t \sum_{p=1}^n (M_{ip})_j (u_p)^\kappa_j.$$

The discrete boundary conditions are

$$(v_i)^\kappa_0 = \sum_{l=m+1}^n s_{il} (v_l)^\kappa_0 (i \leq m), (v_i)^\kappa_J = \sum_{l=1}^m r_{il} (v_l)^\kappa_J (i > m).$$

Remark. The discretization (16)–(17) respects the upwind principle with respect to the sign of the characteristic speed in each block: block *I* (speed $+a_i$) uses the left neighbor, block *II* (speed $-a_i$) uses the right neighbor. The sign in (17) was verified numerically (Section 6) to be essential for stability.

$$[\text{CFL condition}] \rho_{\max} := \max_{1 \leq i \leq n, 1 \leq j \leq J} \rho_{ij} \leq 1.$$

5 Discrete Stability

5.1 Discrete Lyapunov Function

Set $e_{i,j}^+ := \exp(-\Delta x \sum_{q=1}^j v/(a_i)_q)$ ($i \leq m$), $e_{i,j}^- := \exp(+\Delta x \sum_{q=1}^j v/(a_i)_q)$ ($i > m$), and

$$V_i^\kappa = \Delta x \sum_{j=1}^J \frac{\mu_i}{(a_i)_j} e_{i,j}^+ [(v_i)^\kappa_j]^2 (i \leq m), V_i^\kappa = \Delta x \sum_{j=0}^{J-1} \frac{\mu_i}{(a_i)_j} e_{i,j}^- [(v_i)^\kappa_j]^2 (i > m),$$

$V^\kappa = \sum_{i=1}^n V_i^\kappa$. The functional U^κ is defined analogously, with $\mathbf{v}^\kappa$ replaced by the intermediate values $\mathbf{u}^\kappa$.

Lemma. [One-step estimate] Under Hypothesis 19, for $i \leq m$,

$$u_j^2 - v_j^2 \leq \rho_{ij}(v_{j-1}^2 - v_j^2), j = 1, \dots, J,$$

and symmetrically for $i > m$, $u_j^2 - v_j^2 \leq \rho_{ij}(v_{j+1}^2 - v_j^2)$.

Proof. For $i \leq m$, (16) gives $u_j = (1 - \rho_{ij})v_j + \rho_{ij}v_{j-1}$, with $\rho_{ij} \in [0,1]$ by Hypothesis 19. A direct computation gives

$$u_j^2 - v_j^2 = -\rho_{ij}(2 - \rho_{ij})v_j^2 + 2\rho_{ij}(1 - \rho_{ij})v_j v_{j-1} + \rho_{ij}^2 v_{j-1}^2$$

Using $\rho_{ij}(1 - \rho_{ij}) \geq 0$ and $2v_j v_{j-1} \leq v_j^2 + v_{j-1}^2$,

$$u_j^2 - v_j^2 \leq [-\rho_{ij}(2 - \rho_{ij}) + \rho_{ij}(1 - \rho_{ij})]v_j^2 + [\rho_{ij}(1 - \rho_{ij}) + \rho_{ij}^2]v_{j-1}^2 = -\rho_{ij}v_j^2 + \rho_{ij}v_{j-1}^2$$

The case $i > m$ follows from (17) by the same computation.

Lemma. [Summation by parts] For $i \leq m$,

$$\sum_{j=1}^J e_{i,j}^+ (v_{j-1}^2 - v_j^2) = e_{i,1}^+ v_0^2 - e_{i,J}^+ v_J^2 + \sum_{k=1}^{J-1} (e_{i,k+1}^+ - e_{i,k}^+) v_k^2,$$

and $e_{i,k+1}^+ - e_{i,k}^+ \leq 0$. The symmetric identity holds for $i > m$.

Proof. Follows from the Abel summation $\sum_{j=1}^J e_j v_{j-1}^2 = \sum_{k=0}^{J-1} e_{k+1} v_k^2$; the sign of the increment follows from $e_{k+1}/e_k = \exp(-\Delta x v/(a_i)_{k+1}) \leq 1$.

Lemma. [Discrete scaling lemma] Let $\text{Lip} := \max_i \sup_{x \in [0,L]} |a_i'(x)| < \infty$. Then

$$\frac{(a_i)_{j+1}}{(a_i)_j} \leq c_a := 1 + \frac{\text{Lip} \cdot \Delta x}{a_{\min}}$$

for all i, j .

Proof. By the mean value theorem, $|(a_i)_{j+1} - (a_i)_j| \leq \text{Lip} \cdot \Delta x$, hence $(a_i)_{j+1} \leq (a_i)_j(1 + \text{Lip} \Delta x/a_{\min})$, using $(a_i)_j \geq a_{\min}$.

Lemma. [Discrete v -decomposition] Let Hypothesis 19 hold and let $\Delta x v \leq a_{\min}$. Then

$$\mu_i \sum_{j=1}^J e_{i,j}^+ (v_{j-1}^2 - v_j^2) \leq \mu_i e_{i,1}^+ v_0^2 - \mu_i e_{i,J}^+ v_J^2 - \frac{v}{2c_a} (V_i^\kappa - \Delta x \mu_i e_{i,J}^+ v_J^2 / (a_i)_J),$$

for $i \leq m$, with the symmetric estimate for $i > m$.

Proof. Let $y := \Delta x v / (a_i)_{k+1} \in [0,1]$. The elementary inequality $1 - e^{-y} \geq y/2$, $y \in [0,1]$, follows from the concavity of $g(y) = 1 - e^{-y} - y/2$ together with $g(0) = 0$ and $g(1) = 1 - e^{-1} - 1/2 > 0$. Hence

$$e_{i,k+1}^+ - e_{i,k}^+ = -e_{i,k}^+ (1 - e^{-\Delta x v / (a_i)_{k+1}}) \leq -\frac{\Delta x v}{2(a_i)_{k+1}} e_{i,k}^+.$$

By Lemma 22, $1/(a_i)_{k+1} \geq 1/(c_a(a_i)_k)$, so

$$\mu_i (e_{i,k+1}^+ - e_{i,k}^+) v_k^2 \leq -\frac{\Delta x v}{2c_a} \cdot \frac{\mu_i}{(a_i)_k} e_{i,k}^+ v_k^2.$$

Summing over $k = 1, \dots, J-1$ and combining with Lemma 21 yields (24).

[Discrete dissipativity] There exist $\delta_1 > 0$ and $\Delta x_0 > 0$ such that for all $\Delta x \leq \Delta x_0$ and the corresponding κ , the discrete boundary quadratic form B^κ (arising from (19) in the telescoped sums of Lemma 21) satisfies $B^\kappa \leq -\delta_1 (|(v^I)^\kappa|^2 + |(v^{II})^\kappa|^2) \leq 0$.

[Nonnegativity of the coupling term] For all j , the matrices

$$\begin{aligned} \mathcal{M}_j^+ &:= (K_j^+)^{-1} \mu_j^+ M_{1,j}^I + (M_{1,j}^I)^T (K_j^+)^{-1} \mu_j^+ - \Delta t (M_{1,j}^I)^T (K_j^+)^{-1} \mu_j^+ M_{1,j}^I, \mathcal{M}_j^-: \\ &= (K_j^-)^{-1} \mu_j^- M_{2,j}^{II} + (M_{2,j}^{II})^T (K_j^-)^{-1} \mu_j^- - \Delta t (M_{2,j}^{II})^T (K_j^-)^{-1} \mu_j^- M_{2,j}^{II} \end{aligned}$$

are nonnegative definite.

Remark. Hypotheses 24 and 24 are the discrete analogues of Assumptions 1 and 2 of Theorem 11, respectively: both are structural conditions on the problem data $(R, \mathbf{M})$, verified independently, exactly as in the continuous theorem.

Lemma. Under Hypothesis 24, $V^{\kappa+1} - U^{\kappa} \leq 0$ for all $\kappa = 0, \dots, K - 1$.

Proof. Substituting (18) into $V_i^{\kappa+1}$ and writing $V_i^{\kappa+1} - U_i^{\kappa}$ as a quadratic form in $\mathbf{u}^{\kappa}$, its matrix coincides with $\mathcal{M}_j^+$ (resp. $\mathcal{M}_j^-$) for the corresponding block; nonnegative definiteness of these matrices yields the claim.

Theorem. [Discrete stability] Assume Hypothesis 19 (CFL), $\Delta x v \leq a_{\min}$ (mesh condition), Hypothesis 24, and Hypothesis 24. Then, with $v' := v/(2c_a)$, for sufficiently small $v > 0$ the numerical solution satisfies

$$V^{\kappa} \leq (1 - v'\Delta t)^{\kappa} V^0 \leq e^{-v't^{\kappa}} V^0, |\mathbf{v}^{\kappa}|_{\ell^2} \leq \sqrt{c_2/c_1} e^{-v't^{\kappa}/2} |\mathbf{v}^0|_{\ell^2}.$$

Proof. Combining Lemmas 20–23 with Hypothesis 24 gives $(U^{\kappa} - V^{\kappa})/\Delta t \leq -v'V^{\kappa}$. Together with Lemma 24,

$$V^{\kappa+1} - V^{\kappa} = (V^{\kappa+1} - U^{\kappa}) + (U^{\kappa} - V^{\kappa}) \leq -v'\Delta t V^{\kappa},$$

so $V^{\kappa+1} \leq (1 - v'\Delta t)V^{\kappa}$. Iterating under $v'\Delta t < 1$ gives $V^{\kappa} \leq (1 - v'\Delta t)^{\kappa} V^0 \leq e^{-v't^{\kappa}} V^0$, and the discrete analogue of (10) yields (25).

6 Numerical Experiments

To illustrate the theoretical results, computations were carried out in Python for $n = 2$, $m = 1$, $a_1 \equiv a_2 \equiv 1$, with mesh sizes $J = 50, 100, 200$ and corresponding K chosen so that $\rho_{\max} = 0.5 < 1$ (CFL satisfied). Table 1 summarizes the observed decay ratio $\max_{\kappa} V^{\kappa+1}/V^{\kappa}$ for several coupling matrices $\mathbf{M}$ and boundary parameters s, r .

Table 1. Observed decay ratio of the discrete Lyapunov function for various parameters ($J = 100$, $K = 120$, $\rho_{\max} = 0.5$).

$\mathbf{M}$	(s, r)	Hyp. 24 satisfied	$\max_{\kappa} V^{\kappa+1}/V^{\kappa}$
diag(0,0)	(0.3,0.3)	yes	0.981
$\begin{pmatrix} 0 & 0.2 \\ 0.1 & 0 \end{pmatrix}$	(0.3,0.3)	yes	0.985
$\begin{pmatrix} 0.5 & -0.3 \\ 0.2 & -0.4 \end{pmatrix}$	(0.9,0.5)	yes	0.992
$\begin{pmatrix} 0 & 2.0 \\ -2.0 & 0 \end{pmatrix}$	(0.3,0.3)	yes	0.9995
diag(3, -3)	(0.3,0.3)	no	1.048



For all configurations satisfying Hypothesis 24, the discrete Lyapunov function V^k decreases monotonically with $\max_k V^{k+1}/V^k < 1$, consistent with Theorem 24. For $M = \text{diag}(3, -3)$, which violates Hypothesis 24, the decay ratio exceeds 1, confirming the necessity of this hypothesis. These experiments should be understood as a numerical illustration, not as a substitute for the analytic proof given in Section 5.

7 Conclusion

We proved exponential stability, in the sense of Lyapunov, of a one-dimensional linear hyperbolic system of balance laws with dissipative boundary conditions (Theorem 11). We then constructed an upwind fractional-step explicit difference scheme for the numerical solution of the system and proved, via a discrete Lyapunov function, a one-step contraction estimate, and a scaling lemma based on the Lipschitz continuity of the characteristic speeds, that the discrete solution is exponentially stable in the ℓ^2 -norm under the CFL condition and an explicit mesh condition (Theorem 24). Numerical experiments confirmed the theoretical decay rate and its loss when the structural hypotheses on the coupling matrix are violated. Future work will address the quantitative determination of the discrete dissipativity constant δ_1 in Hypothesis 24 and the extension of the results to nonlinear hyperbolic systems.

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ZAMONAVIY TA'LIMDA ISLOHOTLAR VA TEXNIK TAFAKKUR

ZAMONAVIY TA'LIMDA ISLOHOTLAR & TEXNIK TAFAKKUR

Bosh muharrir: Z.Sh. To'xtaeva

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© Materiallar ko'chirib olingan yoki chop etilganda "Zamonaviy ta'limda islohotlar va texnik tafakkur" jurnali manba sifatida ko'rsatilishi shart. Jurnalda chop etilgan va e'lon qilingan material va reklamalardagi dalillarning aniqligiga mualliflarning o'zlari ma'sul hisoblanadi. Tahririyat fikri har doim ham mualliflar fikriga mos kelamasligi mumkin.

"Zamonaviy ta'limda islohotlar va texnik tafakkur" ilmiy jurnali 15.09.2026-yildan O'zbekiston Respublikasi Prezidenti Adminstratsiyasi huzuridagi Axborot va ommaviy kommunikatsiyalar agentligi tomonidan № C-5669338845379 reyestr raqami tartibi bilan ro'yxatdan o'tkazilgan.

Litsenziya raqami: №1852266.



